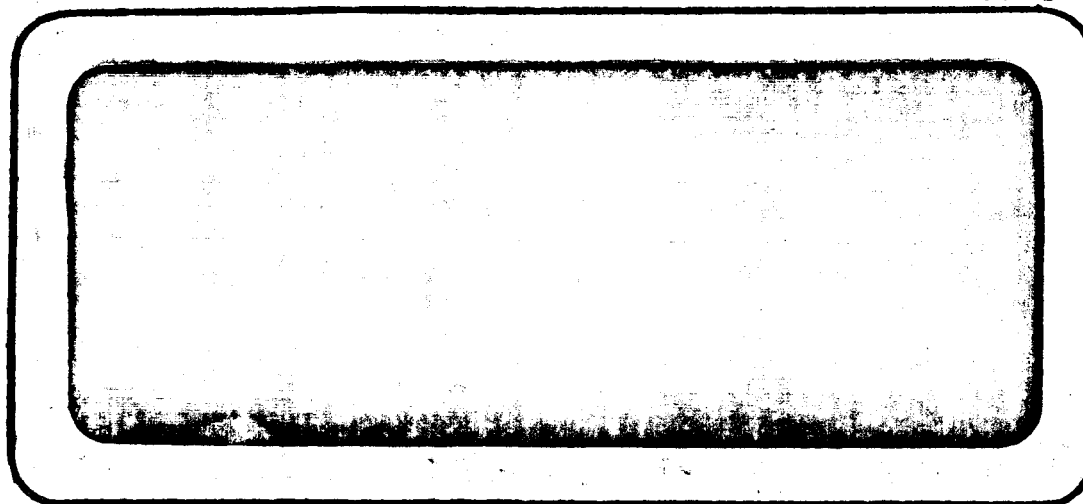


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THE CRITICAL CRACK LENGTH

IN

PRESSURIZED, MONOCOQUE CYLINDERS

by

E. E. Sechler and M. L. Williams

September 1959

Guggenheim Aeronautical Laboratory

California Institute of Technology

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E. E. Sechler* and M. L. Williams**

Abstract

The use of a theoretical solution based on an elastically supported strip adjacent to a crack in a monocoque cylinder under pressure leads to the determination of the critical parameters of the problem. These are a) the pressure parameter $\lambda = \frac{p}{E} \left(\frac{R}{t}\right)^2$ and b) the crack parameter l_{cr}/\sqrt{Rt} . Two unknown factors C_1 and C_2 representing the amount of elastic support and the sensitivity of the material to crack growth respectively must be determined experimentally. When this is done, use of the derived equations leads to critical crack lengths which are in reasonable agreement with experimental results.

*Professor of Aeronautics, California Institute of Technology

**Associate Professor of Aeronautics, California Institute of Technology.

Introduction

The problem of the critical crack length in a pressurized cylinder is of great importance to the aircraft and missile industry. The critical crack length is defined as that length of crack or slot in the skin of a cylindrical shell that will be self-propagating under the stress field in the shell. Any crack length smaller than critical will only cause loss of pressure through leakage.

Analysis of the Problem

The basic parameters of the problem may be obtained by considering an element of the shell near the region of the crack. See Figure 1. According to Hetenyi (Ref. 1, page 31) a longitudinal element of a cylindrical tube loaded symmetrically with respect to its axis can be regarded as a beam on an elastic foundation with the foundation modulus being given by

$$k = \frac{Et}{R^2} \quad (1)$$

It appears reasonable to assume that, when a crack is formed, the above modulus is reduced but that the form of the modulus remains essentially the same. Thus, we will assume, for the case of a cylinder with a crack, that the element adjacent to the crack acts as a beam on an elastic foundation with a foundation modulus equal to

$$k' = \frac{C_1 Et}{R^2} = C_1 k \quad (2)$$

See Figure 2. Obviously the support from the cylinder is greater near the ends of the crack than it is in the center of the cracked area and, as the crack length increases, the average amount of support (represented by k') may decrease. We may thus find that C_1 is dependent upon the crack length ℓ to some power

Consider now an element adjacent to the crack under a pressure loading, p , an axial loading N_x , and on an elastic foundation with a modulus of k' . See Figure 3. Assume that the boundary conditions are zero slope and zero deflection at the ends of the element. Then from equation (109) of Ref. 1, we find the deflection is given by

$$\eta_A = \frac{p}{k'} \left[1 - \frac{1}{\rho \sinh \alpha \ell + \alpha \sin \rho \ell} \left\{ \alpha \cosh \alpha x \sin \rho (\ell - x) + \rho \sinh \alpha x \cos \rho (\ell - x) + \alpha \sin \rho x \cosh \alpha (\ell - x) + \rho \cos \rho x \sinh \alpha (\ell - x) \right\} \right] \quad (3)$$

where

$$\alpha = \sqrt{\frac{k'}{4EI} + \frac{N_x}{4EI}} \quad (4)$$

$$\beta = \sqrt{\frac{k'}{4EI} - \frac{N_x}{4EI}} \quad (5)$$

The above equation holds for $N_x \leq 2\sqrt{k'EI}$. For values of $N_x > 2\sqrt{k'EI}$ the equation (3) becomes

$$\eta_B = \frac{p}{k'} \left[1 - \frac{1}{\bar{\beta} \sinh \alpha l + \alpha \sinh \bar{\beta} l} \left\{ \alpha \cosh \alpha x \sinh \bar{\beta} (l-x) + \bar{\beta} \sinh \alpha x \cosh \bar{\beta} (l-x) + \alpha \sinh \bar{\beta} x \cosh \alpha (l-x) + \bar{\beta} \cosh \bar{\beta} x \sinh \alpha (l-x) \right\} \right] \quad (6)$$

where

$$\bar{\beta} = \sqrt{\frac{N_x}{4EI} - \frac{k'}{4EI}} \quad (7)$$

and α remains the same as previously defined. Differentiating equations (3) and (4) twice we obtain the expressions for slope and moment for the two cases, namely,

$$\frac{d\eta_A}{dx} = \theta_A = -\frac{p}{k'} A \left[(\alpha^2 + \beta^2) \sinh \alpha x \sin \beta (l-x) - (\alpha^2 + \beta^2) \sin \beta x \sinh \alpha (l-x) \right] \quad (8)$$

$$EI \frac{d^2 \eta_A}{dx^2} = M_A = -\frac{pEI}{k'} A (\alpha^2 + \beta^2) \left[\alpha \cosh \alpha x \sin \beta (l-x) - \beta \sinh \alpha x \cos \beta (l-x) - \beta \cos \beta x \sinh \alpha (l-x) + \alpha \sin \beta x \cosh \alpha (l-x) \right] \quad (9)$$

$$\text{where } A = \frac{1}{\bar{\beta} \sinh \alpha l + \alpha \sinh \bar{\beta} l} \quad (10)$$

The maximum moment at the ends ($x = \ell$) is

$$M_{A_m} = - \frac{pEI}{k'} \frac{\alpha \sinh \beta \ell - \beta \sinh \alpha \ell}{\alpha \sinh \beta \ell + \beta \sinh \alpha \ell} (\alpha^2 + \beta^2) \quad (11)$$

for $N_x < 2 \sqrt{k'EI}$, and

$$\frac{d\eta_B}{dx} = \theta_B = - \frac{p}{k'} B(\alpha^2 - \beta^2) \left[\sinh \alpha x \sinh \bar{\beta}(\ell - x) - \sinh \bar{\beta} x \sinh \alpha(\ell - x) \right] \quad (12)$$

$$EI \frac{d^2 \eta_B}{dx^2} = M_B = - \frac{pEI}{k'} B(\alpha^2 - \beta^2) \left[\alpha \cosh \alpha x \sinh \bar{\beta}(\ell - x) - \bar{\beta} \sinh \alpha x \cosh \bar{\beta}(\ell - x) \right. \\ \left. - \bar{\beta} \cosh \bar{\beta} x \sinh \alpha(\ell - x) + \alpha \sinh \bar{\beta} x \cosh \alpha(\ell - x) \right] \quad (13)$$

where

$$B = \frac{1}{\bar{\beta} \sinh \alpha \ell + \alpha \sinh \bar{\beta} \ell} \quad (14)$$

The maximum moment at the ends ($x = \ell$) is

$$M_{B_m} = - p \frac{EI}{k'} \frac{\alpha \sinh \bar{\beta} \ell - \bar{\beta} \sinh \alpha \ell}{\alpha \sinh \bar{\beta} \ell + \bar{\beta} \sinh \alpha \ell} (\alpha^2 - \beta^{-2}) \quad (15)$$

for $N_x > 2 \sqrt{k'EI}$

from equations (4) and (5)

$$\alpha^2 + \beta^2 = 2 \sqrt{\frac{k'}{4EI}} \quad (16)$$

and, from equations (4) and (7)

$$\alpha^2 - \bar{\beta}^2 = 2\sqrt{\frac{k'}{4EI}} \quad (17)$$

Replacing EI by the plate bending stiffness

$$D = \frac{Et^3}{12(1 - \nu^2)}$$

and k' by equation (2)

$$\alpha^2 + \beta^2 = \alpha^2 - \bar{\beta}^2 = 2 \frac{\sqrt{3C_1(1 - \nu^2)}}{Rt} \quad (18)$$

Thus, the end moment equations become

$$M_{A_{in}} = -p \frac{Rt}{2 \sqrt{3C_1(1 - \nu^2)}} \left[\frac{\alpha \sin \beta \ell}{\alpha \sin \beta \ell} - \frac{\bar{\beta} \sinh \alpha \ell}{\bar{\beta} \sinh \alpha \ell} \right] \quad (19)$$

$$M_{B_{in}} = -p \frac{Rt}{2 \sqrt{3C_1(1 - \nu^2)}} \left[\frac{\alpha \sinh \bar{\beta} \ell}{\alpha \sinh \bar{\beta} \ell} - \frac{\bar{\beta} \sinh \alpha \ell}{\bar{\beta} \sinh \alpha \ell} \right] \quad (20)$$

Also, from equation (2) and from the fact that

$$N_x = \frac{pR}{2} \quad (21)$$

for a simply pressurized cylinder (no external loads)

$$\alpha = \frac{\sqrt[4]{3C_1(1 - \nu^2)}}{\sqrt{Rt}} \sqrt{1 + \frac{p}{E} \left(\frac{R}{t}\right)^2 \frac{\sqrt{3C_1(1 - \nu^2)}}{2C_1}} \quad (22)$$

$$\beta = \frac{\sqrt[4]{3C_1(1 - \nu^2)}}{\sqrt{Rt}} \sqrt{1 - \frac{p}{E} \left(\frac{R}{t}\right)^2 \frac{\sqrt{3C_1(1 - \nu^2)}}{2C_1}} \quad (23)$$

-6-

$$\bar{\beta} = \frac{\sqrt[4]{3C_1(1-\nu^2)}}{\sqrt{Rt}} \sqrt{\frac{p}{E} \left(\frac{R}{t}\right)^2 \frac{\sqrt{3C_1(1-\nu^2)}}{2C_1} - 1} \quad (24)$$

setting $\frac{p}{E} \left(\frac{R}{t}\right)^2 = \lambda$ (25)

and

$$\sqrt{3C_1(1-\nu^2)} = \delta^2 \quad (26)$$

$$\alpha = \frac{\delta}{\sqrt{Rt}} \sqrt{1 + \frac{\lambda \delta^2}{2C_1}} \quad (27)$$

$$\mu = \frac{\delta}{\sqrt{Rt}} \sqrt{1 - \frac{\lambda \delta^2}{2C_1}} \quad (28)$$

$$\bar{\beta} = \frac{\delta}{\sqrt{Rt}} \sqrt{\frac{\lambda \delta^2}{2C_1} - 1} \quad (29)$$

and equations (19) and (20) become

$$M_{A_m} = -p \frac{Rt}{2\delta^2} \left[\frac{\sqrt{1 + \frac{\lambda \delta^2}{2C_1}} \sin \frac{\delta \ell}{\sqrt{Rt}} \sqrt{1 - \frac{\lambda \delta^2}{2C_1}} - \sqrt{1 - \frac{\lambda \delta^2}{2C_1}} \sinh \frac{\delta \ell}{\sqrt{Rt}} \sqrt{1 + \frac{\lambda \delta^2}{2C_1}}}{\sqrt{1 + \frac{\lambda \delta^2}{2C_1}} \sin \frac{\delta \ell}{\sqrt{Rt}} \sqrt{1 - \frac{\lambda \delta^2}{2C_1}} + \sqrt{1 - \frac{\lambda \delta^2}{2C_1}} \sinh \frac{\delta \ell}{\sqrt{Rt}} \sqrt{1 + \frac{\lambda \delta^2}{2C_1}}} \right] \quad (30)$$

$$= -p \frac{Rt}{2\delta^2} [\Delta]$$

$$M_{B_m} = -p \frac{Rt}{2\delta^2} \left[\frac{\sqrt{1 + \frac{\lambda \delta^2}{2C_1}} \sinh \frac{\delta \ell}{\sqrt{Rt}} \sqrt{\frac{\lambda \delta^2}{2C_1} - 1} - \sqrt{\frac{\lambda \delta^2}{2C_1} - 1} \sinh \frac{\delta \ell}{\sqrt{Rt}} \sqrt{1 + \frac{\lambda \delta^2}{2C_1}}}{\sqrt{1 + \frac{\lambda \delta^2}{2C_1}} \sinh \frac{\delta \ell}{\sqrt{Rt}} \sqrt{\frac{\lambda \delta^2}{2C_1} - 1} + \sqrt{\frac{\lambda \delta^2}{2C_1} - 1} \sinh \frac{\delta \ell}{\sqrt{Rt}} \sqrt{1 + \frac{\lambda \delta^2}{2C_1}}} \right] \quad (31)$$

$$= -p \frac{Rt}{2\delta^2} [\Sigma]$$

The bending stress due to these moments are

$$\sigma_A = \pm \frac{6M_{A_m}}{t^2} \quad (32)$$

$$\sigma_B = \pm \frac{6M_{B_m}}{t^2} \quad (33)$$

and the total stress in the axial direction is therefore

$$\sigma_{A_T} = \pm \frac{3p}{\delta^2} \left(\frac{R}{t} \right) [A] + \frac{p}{2} \left(\frac{R}{t} \right) = p \left(\frac{R}{t} \right) \left\{ \pm \frac{3}{\delta^2} [A] + \frac{1}{2} \right\} = \sigma_{1_A} \quad (34)$$

or

$$\sigma_{B_T} = p \left(\frac{R}{t} \right) \left\{ \pm \frac{3}{\delta^2} [B] + \frac{1}{2} \right\} = \sigma_{1_B} \quad (35)$$

The circumferential stress at the edge of the crack is given by

$$\sigma_\theta = C_2 p \frac{R}{t} = \sigma_2 \quad (36)$$

where C_2 is a stress concentration factor representing the sensitivity of the particular material to a stress concentration. The value of C_2 will therefore be different for different materials whereas C_1 can be expected to be essentially a constant insofar as materials are concerned.

Using the octahedral shear theory of failure, the maximum value of the octahedral shear is given by

$$\tau_n = \frac{1}{3} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \quad (37)$$

where $\sigma_3 = 0$ or

$$\tau_n = \frac{\sqrt{2}}{3} \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2} \quad (38)$$

and this must be related to the ultimate strength of the material, thus

$$C'_3 \sigma_u = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2} \quad (39)$$

where C'_3 includes the constants relating the shearing stress to the ultimate tensile strength and also includes a factor covering the notch sensitivity of the material since it is directly dependent upon the value of C_2 .

For the cylinder without a flaw

$$\sigma_1 = \frac{p_o}{2} \left(\frac{R}{t} \right) \quad (40)$$

$$\sigma_2 = p_o \left(\frac{R}{t} \right) \quad (41)$$

and

$$C''_3 \sigma_u = p_o \left(\frac{R}{t} \right) \quad (42)$$

Combining the constants C'_3 and C''_3 into C_3 we finally find from equations (34), (39), and (42)

$$C_3 p_o = p_A \sqrt{\left\{ \pm \frac{3}{82} \left[A \right] + \frac{1}{2} \right\}^2 + C_2^2 - C_2 \left\{ \pm \frac{3}{82} \left[A \right] + \frac{1}{2} \right\}} \quad (43)$$

or

$$C_3 p_o = p_B \sqrt{\left\{ \pm \frac{3}{\delta^2} \left[B \right] + \frac{1}{Z} \right\}^2 + C_2^2 - C_2 \left\{ \pm \frac{3}{\delta^2} \left[B \right] + \frac{1}{Z} \right\}} \quad (44)$$

The sign to be used for the term $\pm \frac{3}{\delta^2} \left[\begin{smallmatrix} A \\ \text{or} \\ B \end{smallmatrix} \right]$ is that which gives the largest value of $C_3 p_o$. Since the bracketed quantities, $\left[A \right]$ and $\left[B \right]$ contain p_A and p_B respectively it is impossible to solve directly for p_A/p_o or p_B/p_o but a solution can be obtained by a successive approximation procedure.

At $l/\sqrt{Rt} = 0$ $\left[\begin{smallmatrix} A \\ \text{or} \\ B \end{smallmatrix} \right] = 0$ and $C_3 = \sqrt{0.25 + C_2^2 - 0.50 C_2}$
and we obtain, finally

$$\frac{p}{p_o} = \frac{\sqrt{0.25 + C_2^2 - 0.50 C_2}}{\sqrt{\left\{ \pm \frac{3}{\delta^2} \left[\begin{smallmatrix} A \\ \text{or} \\ B \end{smallmatrix} \right] + \frac{1}{Z} \right\}^2 + C_2^2 - C_2 \left\{ \pm \frac{3}{\delta^2} \left[\begin{smallmatrix} A \\ \text{or} \\ B \end{smallmatrix} \right] + \frac{1}{Z} \right\}}} = \frac{\sigma}{\sigma_o} \quad (45)$$

where σ and σ_o are calculated by the usual simple thin walled pressure vessel equation

$$\sigma = \frac{pR}{t} \quad \sigma_o = \frac{p_o R}{t} \quad (46)$$

Experimental Verification

Three sets of experimental data were available for checking the above theoretical treatment. At GALCIT a series of experiments were carried out using brass shim stock of 0.001 in. and 0.005 in. thickness formed into cylinders of 3.0 in. and 5.0 in. diameter. The cylinders had a 5 in. free length between brass end plates which were soldered on. Loading was done with water pressure.

An inverse process was used in which a crack of definite length was cut into the cylinder with a sharp knife and then increasing pressure was applied until failure occurred. The cracks were all parallel to the cylinder axis. Each different sheet of brass shim stock was statically tested to determine its strength properties and the ratio of failing stresses σ_f/σ_u was based on the actual ultimate failing stress of the material. The value of σ_f is the failing hoop stress with a crack and the value of σ_u is the ultimate stress of the material as determined by the failing hoop stress without a crack. An average value of $E = 15 \times 10^6$ psi was used for the brass material. Tables 1, 2, and 3 and Figure 4 indicate the results of the tests in brass. Using average experimental values of σ_f/σ_u as a function of ℓ_{cr}/\sqrt{Rt} and $\lambda = \frac{p}{E} \left(\frac{R}{t}\right)^2$ average values of $C_1 = 0.10$ and $C_2 = 2.50$ were obtained. These values were then used to plot the theoretical curves shown in Figure 4.

As can be seen in Figure 4, essentially all of the experimental points lie between the curves for $\lambda = 2.0$ and $\lambda = 8.0$ which represent the experimental values of λ . Also that, in general, for a given value of ℓ_{cr}/\sqrt{Rt} , higher values of λ correspond to higher values of σ_f/σ_u as predicted by the theory.

The other two sets of data originate from NACA Technical Note 3993 and concern cylinders made from 2024 and 7075 sheet material. The results of these tests and the comparison with the theory are shown in Tables 4 and 5 and Figures 5 and 6 respectively. Actual test values of ultimate stress were not available and, therefore, nominal values were used throughout. As is well known, ultimate tensile

strengths differ considerably with different thicknesses of sheet stock and would, therefore, lead to scatter in the test results.

Calculating the values of C_1 and C_2 from average test data points it was found that C_1 was essentially constant at a value of $C_1 = 0.10$ (the same as for the brass cylinders) and that $C_2 = 1.0$ for the 2024 material and $C_2 = 0.75$ for the 7075 material.

Again, the test data show the same trends as that predicted by the approximate theoretical treatment and, while the scatter is considerably higher than that for the brass cylinders, this is explained by the use of an average nominal ultimate tensile stress for the materials.

Two additional sets of data relating to the angle of the crack with respect to the cylinder axis are of interest. These are shown in Tables 6 and 7 and relate to brass shim stock cylinders tested at GALCIT and 2024 sheet cylinders tested at the NASA. These data indicate that the use of an effective crack length given by

$$l_e = l_{cr} \cos \theta$$

gives results which lie well within the scatter band of the experimental data for axial cracks (Figures 4 and 5) even up to angles of $\theta = 60^\circ$.

Conclusions and Discussion

The approximate theoretical treatment based on an elastic support of the material near the crack appears to give reasonable results when checked with experiment. For the two aluminum alloys 2024 and 7075 a more accurate determination of C_1 and C_2 would be

advisable and this could be done by an accurate determination of the ultimate strengths of the materials to be used for the cylinder tests.

For new materials, such as stainless steels, it will be necessary to establish the proper values of C_1 and C_2 by a limited number of actual cylinder tests using a range of values of both ℓ_{cr}/\sqrt{Rt} and λ after which, the complete family of curves can be drawn by use of the theoretical equations. To facilitate this work, Figures 7, 8, and 9 are presented. These figures give graphical solutions for the values $\left[A \right]$ and $\left[B \right]$ as functions of $\delta\ell/\sqrt{Rt}$ and $\lambda\delta^2/2C_1$ and values of σ_f/σ_u as functions of $\left[A \right]$ or $\left[B \right]$.

References

1. Hetenyi, M.: "Beams on Elastic Foundation", University of Michigan Press, 1946.
2. Peters, R. W. and Kuhn, P.: "Bursting Strength of Unstiffened Pressure Cylinders with Slits. NACA Technical Note No. 3993, 1957.

TABLE I

All specimens 3.0" diameter and 0.001"

thick of brass shim stock

$$E = 15 \times 10^6 \text{ psi}$$

No.	Crack length ℓ_{cr}	Failing pressure P_f psi	P_f/P_{ult} $= \sigma_f/\sigma_u$	$\frac{\ell_{cr}}{\sqrt{Rt}}$	$\lambda = \frac{p}{E} \left(\frac{R}{t} \right)^2$
1	0	38.5	1.000	0	5.77
2	0	38.0	1.000	0	5.70
3	0	38.5	1.000	0	5.77
4	0.060	33.8	0.884	1.58	5.07
5	0.080	30.4	0.795	2.12	4.56
6	0.130	24.8	0.649	3.42	3.72
7	0.200	20.1	0.525	5.26	3.18
8	0.200	20.0	0.523	5.26	3.00
9	0.250	17.0	0.445	6.58	2.55
10	0.250	17.2	0.450	6.58	2.58
11	0.260	18.0	0.470	6.84	2.70
12	0.300	15.8	0.414	7.89	2.37
13	0.300	14.5	0.379	7.89	2.17
14	0.350	12.8	0.335	9.22	1.92
15	0.420	12.8	0.335	11.05	1.92
16	0.500	9.5	0.248	13.15	1.43
17	0.600	8.9	0.233	15.80	1.34
18	0.700	7.4	0.194	18.42	1.11

TABLE 2

All specimens 5.0" diameter and 0.001"

thick of brass shim stock

$E = 15 \times 10^6$ psi

No.	Crack length l_{cr}	Failing pressure P_f psi	P_f/P_{ult} $= \sigma_f/\sigma_u$	$\frac{l_{cr}}{\sqrt{Rt}}$	$\lambda = \frac{P}{E} \left(\frac{R}{t} \right)^2$
1	0	21.5	1.000	0	8.95
2	0.063	19.2	0.892	1.26	8.00
3	0.063	18.5	0.860	1.26	7.70
4	0.125	16.0	0.744	2.50	6.66
5	0.187	14.9	0.698	3.74	6.20
6	0.187	15.5	0.720	3.74	6.46
7	0.200	14.0	0.622	4.00	5.83
8	0.250	12.9	0.600	5.00	5.37
9	0.250	12.1	0.563	5.00	5.04
10	0.300	11.0	0.511	6.00	4.58
11	0.312	10.5	0.489	6.24	4.38
12	0.312	10.8	0.502	6.24	4.50

TABLE 3

All specimens 5.0" diameter and 0.003"
thick of brass shim stock
 $E = 15 \times 10^6$ psi

No.	Crack length l_{cr}	Failing pressure P_f psi	P_f/P_{ult} $= \sigma_f/\sigma_u$	$\frac{l}{\sqrt{Rt}}$	$\lambda = \frac{P}{E} \left(\frac{R}{t} \right)^2$
1	0	76.0	1.000	0	3.52
2	0.095	67.6	0.890	1.11	3.13
3	0.195	59.9	0.775	2.27	2.78
4	0.230	58.0	0.764	2.68	2.68
5	0.250	51.0	0.671	2.91	2.36
6	0.250	54.0	0.711	2.91	2.50
7	0.250	52.0	0.684	2.91	2.41
8	0.295	49.6	0.654	3.43	2.30

TABLE 4

From NACA Technical Note 3993

Material-2024-T3

Assumed $\sigma_u = 67,000$ psi transverse (specs. 1-20 and 50-54 inc.)

$\sigma_u = 68,000$ psi longitudinal (specs. 30-32 inc.)

$E = 10 \times 10^6$ psi

No.	Spec. length L(ins)	Spec. diameter D(ins)	Sheet thickness t(ins)	Crack length ℓ_{cr} (ins)	Failing stress σ_f ksi	$\frac{\sigma_f}{\sigma_u}$	$\frac{\ell_{cr}}{\sqrt{Rt}}$	$\lambda = \frac{P}{E} \left(\frac{R}{t} \right)^2$
1	20	7.2	0.012	0.24	40.2	0.60	1.16	1.21
2	20	7.2	0.012	0.24	41.4	0.62	1.16	1.24
3	20	7.2	0.012	0.50	30.6	0.46	2.40	0.92
4	20	7.2	0.012	0.50	26.4	0.39	2.40	0.79
5	20	7.2	0.012	0.96	19.9	0.30	4.62	0.60
6	20	7.2	0.012	0.96	20.4	0.30	4.62	0.61
7	40	7.2	0.012	0.96	21.0	0.31	4.62	0.63
8	20	7.2	0.012	1.87	10.7	0.16	8.99	0.32
9	20	7.2	0.012	3.82	4.9	0.07	18.23	0.15
10	40	7.2	0.012	7.69	2.6	0.04	37.03	0.073
11	20	7.2	0.015	0.62	29.3	0.44	2.67	0.70
12	20	7.2	0.015	1.20	17.2	0.26	5.16	0.41
13	20	7.2	0.015	2.40	8.7	0.13	10.32	0.21
14	20	7.2	0.025	1.03	21.9	0.33	3.44	0.32
15	20	7.2	0.025	2.00	10.4	0.16	6.67	0.15
16	20	7.2	0.025	5.63	3.4	0.05	18.76	0.049
17	20	7.2	0.006	0.24	44.4	0.66	1.63	2.66

TABLE 4 (Cont'd)

E = 10×10^6 psi

No.	Spec. length L(ins)	Spec. diameter D(ins)	Sheet thickness t(ins)	Crack length ℓ_{cr} (ins)	Failing stress σ_f ksi	$\frac{\sigma_f}{\sigma_u}$	$\frac{\ell_{cr}}{\sqrt{Rt}}$	$\lambda = \frac{P(R)}{E(t)^2}$
18	20	7.2	0.006	0.47	27.3	0.41	3.20	1.64
19	20	7.2	0.006	0.96	20.7	0.31	6.53	1.24
20	20	7.2	0.006	1.96	10.2	0.15	13.35	0.61
30*	20	7.2	0.012	0.96	22.8	0.34	4.62	0.68
31*	20	7.2	0.012	1.91	10.2	0.15	9.21	0.31
32*	20	7.2	0.012	3.83	5.4	0.08	18.42	0.16
50	74.5	28.8	0.015	0.63	41.3	0.62	1.36	3.96
51	74.5	28.8	0.015	1.28	29.8	0.44	2.76	2.86
52	74.5	28.8	0.015	2.55	20.4	0.30	5.49	1.96
53	74.5	28.8	0.015	5.10	11.3	0.17	10.97	1.08
54	60	28.8	0.015	7.70	8.2	0.12	16.59	0.79

*Sheet grain running circumferentially. In all other specimens, sheet grain in axial direction.

TABLE 5

From NACA Technical Note 3993

Material 7075-T6

Assumed $\sigma_u = 78,000$ psi $E = 10 \times 10^6$ psi

No.	Spec. length L(ins)	Spec. diameter D(ins)	Sheet thickness t(ins)	Crack length ℓ_{cr} (ins)	Failing stress σ_f ksi	$\frac{\sigma_f}{\sigma_{ult}}$	$\frac{\ell}{\sqrt{Rt}}$	$\lambda = \frac{p}{E} \left(\frac{R}{t} \right)^2$
21	20	7.2	0.016	0.66	19.4	0.25	2.74	0.44
22	20	7.2	0.016	1.29	11.7	0.15	5.37	0.26
23	20	7.2	0.016	2.56	5.5	0.07	10.66	0.12
24	20	7.2	0.025	1.00	16.6	0.21	3.34	0.24
25	20	7.2	0.025	2.02	8.4	0.11	6.74	0.12
26	20	7.2	0.025	4.00	3.7	0.05	13.33	0.05
55	74.5	28.8	0.016	0.62	37.2	0.48	1.29	3.35
56	74.5	28.8	0.016	1.30	24.9	0.32	2.71	2.24
57	60	28.8	0.016	2.58	15.6	0.20	5.37	1.40
58	60	28.8	0.016	5.16	8.5	0.11	10.76	0.77

TABLE 6

Brass shim stock cylinders

Constant crack length, $\ell_{cr} = 0.125"$

All specimens 5.0" diameter and 0.001" thick

$E = 15 \times 10^6$ psi

No.	Crack angle θ	Effective crack ℓ_e (ins)	Test P_f/P_u $= \sigma_f/\sigma_u$	$\frac{\ell E}{\sqrt{Rt}}$	Average P_f/P_u $= \sigma_f/\sigma_u$	$\lambda = \frac{P}{E} \left(\frac{R}{t} \right)^2$
1	0	0.125	0.763	2.50	0.76	6.83
2	15.0	0.121	0.800	2.42		7.17
3	15.0	0.121	0.800	2.42	0.80	7.17
4	22.5	0.116	0.823	2.32		7.38
5	22.5	0.116	0.788	2.32	0.81	7.04
6	30.0	0.108	0.824	2.16		7.38
7	30.0	0.108	0.833	2.16	0.80	7.46
8	30.0	0.108	0.740	2.16		6.62
9	45.0	0.0885	0.837	1.77		7.50
10	45.0	0.0885	0.847	1.77		7.58
11	45.0	0.0885	0.876	1.77	0.87	7.83
12	45.0	0.0885	0.904	1.77		8.08
13	60	0.0625	0.851	1.25		7.63
14	60	0.0625	0.884	1.25	0.87	7.92
15	67.5	0.0477	0.890	0.96		7.96
16	67.5	0.0477	0.961	0.96	0.93	8.63
17	75	0.0324	0.935	0.65		8.38
18	75	0.0324	0.930	0.65	0.93	8.33

TABLE 7

From NACA Technical Note 3993

Material 2024-T3

Assumed $\sigma_u = 67,000$ psi

$D = 7.2''$, $t = 0.012''$

$E = 10 \times 10^6$ psi

No.	Crack angle θ°	Crack length l_{cr} (ins)	Effective crack length l_E	Test σ_f / σ_u	$\frac{l_E}{\sqrt{Rt}}$	$\lambda = \frac{P}{E} \left(\frac{R}{t} \right)^2$
33	30	0.47	0.41	0.55	1.92	1.10
34	30	0.95	0.82	0.34	3.95	0.68
35	30	7.70	6.67	0.04	32.08	0.08
36	45	0.49	0.35	0.60	1.68	1.21
37	45	0.97	0.69	0.42	3.32	0.84
38	45	1.93	1.36	0.26	6.55	0.52
39	45	3.84	2.71	0.12	13.04	0.42
40	45	7.75	5.48	0.03	26.36	0.06
41	60	0.45	0.23	0.64	1.10	1.29
42	60	0.98	0.49	0.51	2.36	1.03
43	60	1.95	0.97	0.37	4.67	0.74
44	60	5.00	2.50	0.12	12.03	0.24

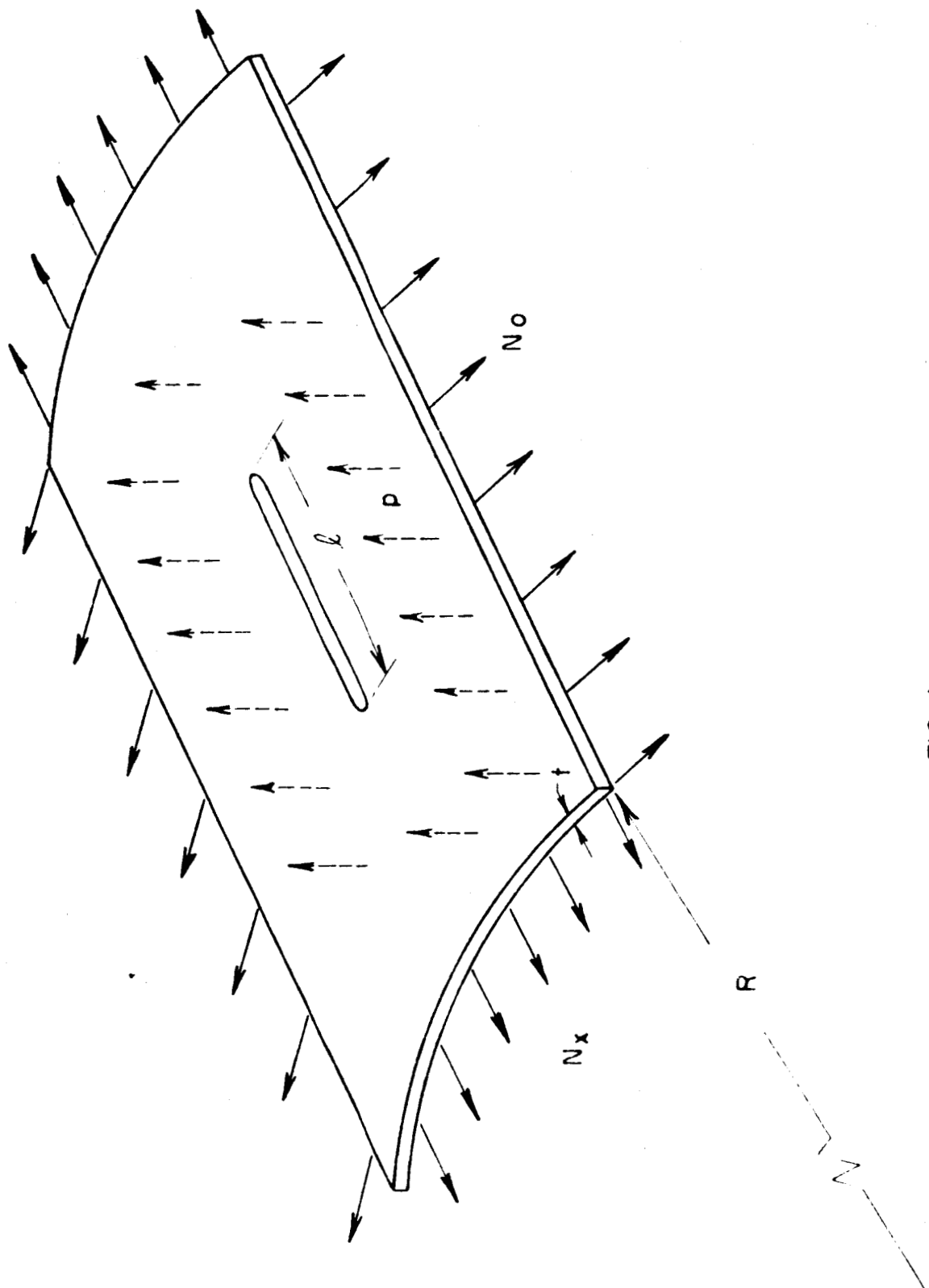


FIG. 1

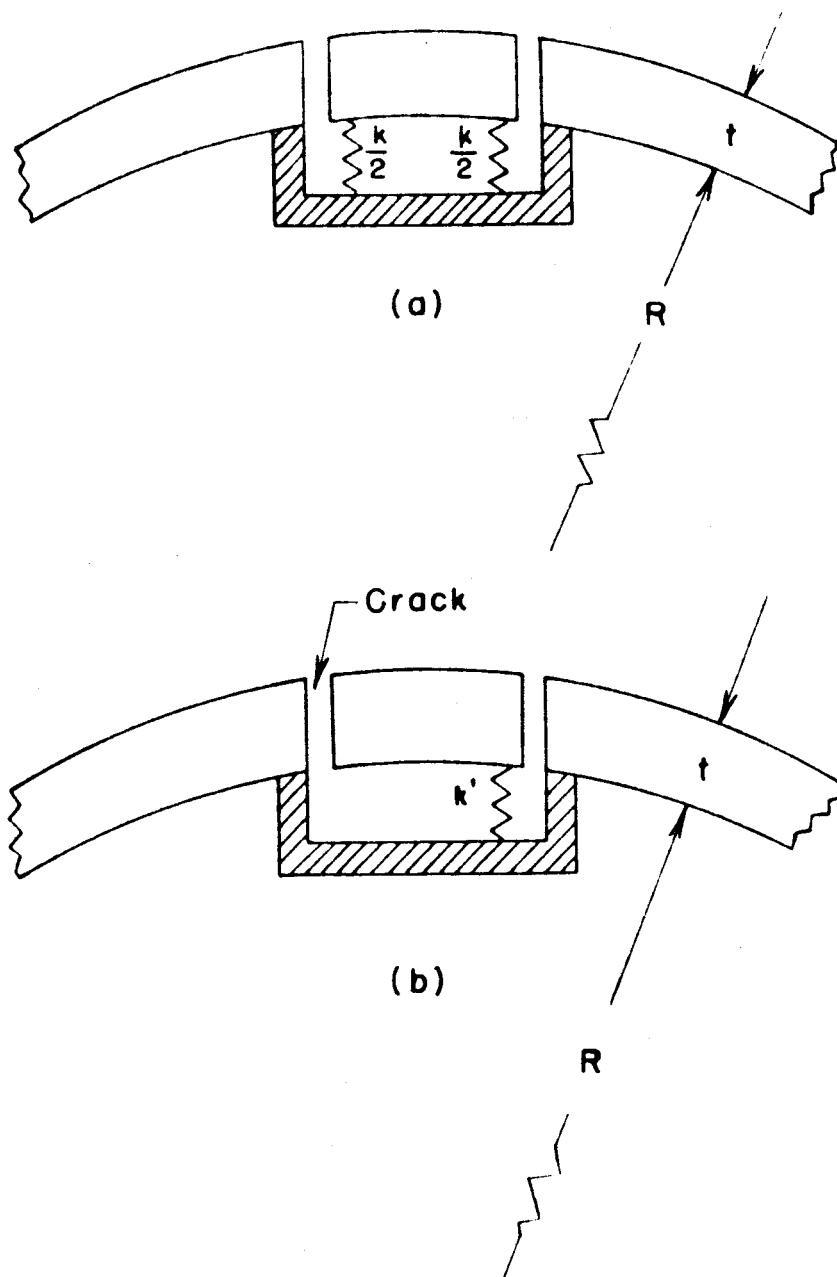


FIG. 2

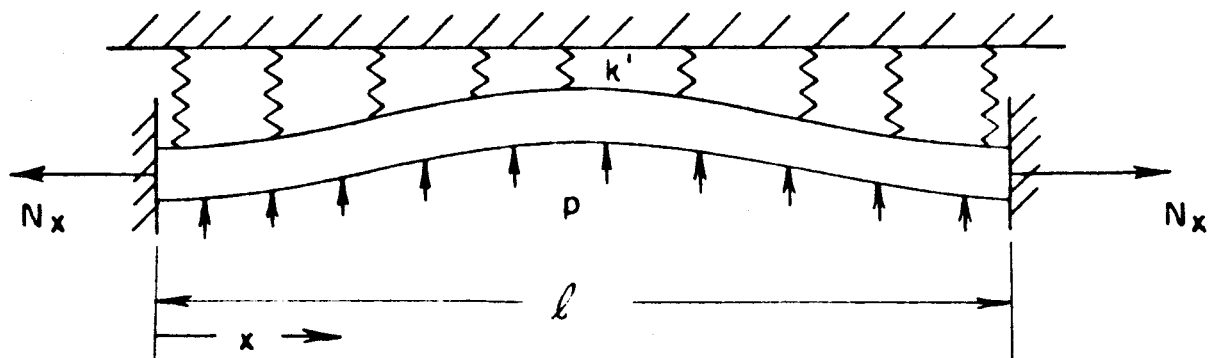


FIG. 3

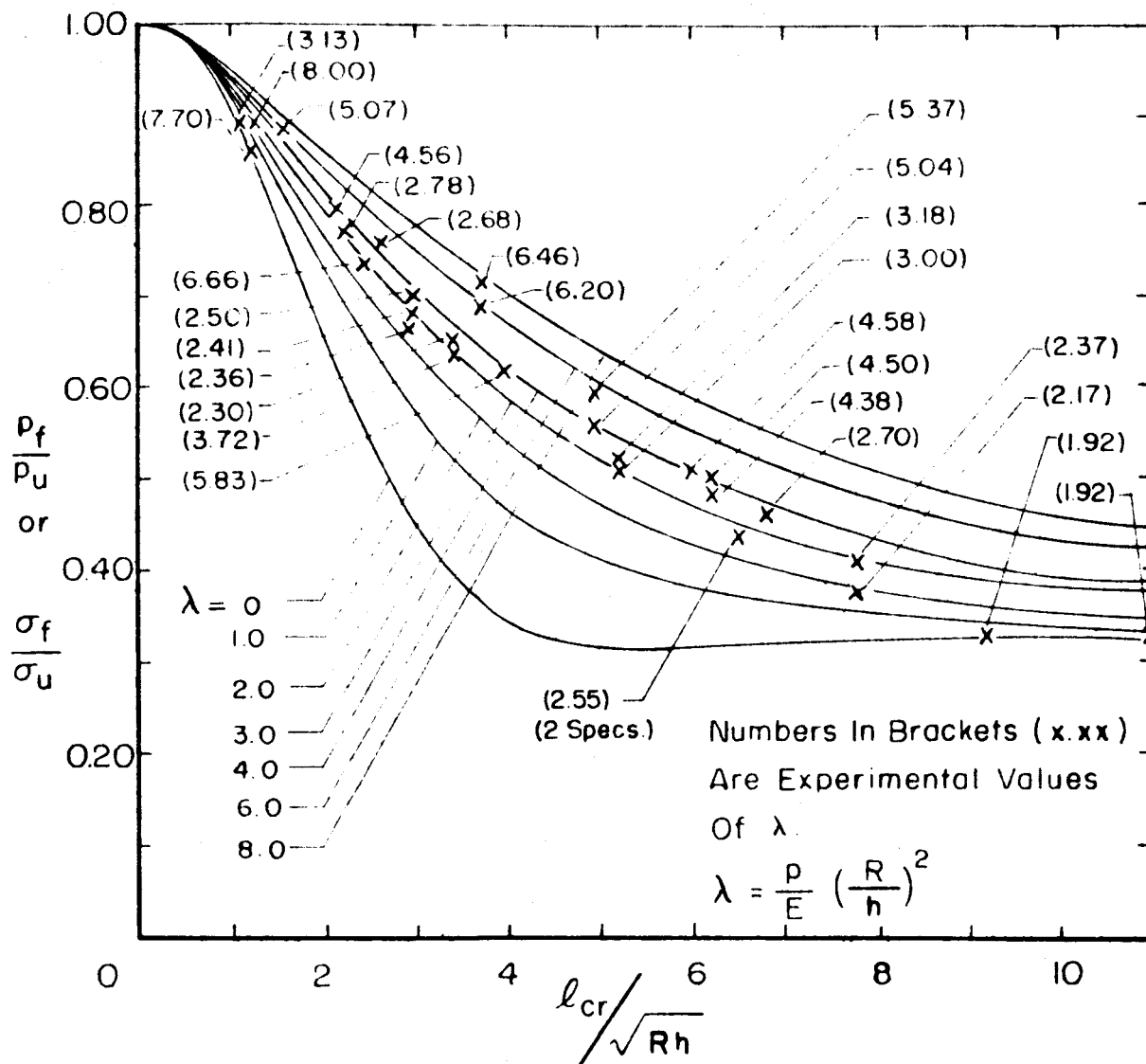


FIG. 4 — COMPARISON OF THEORY AND EXPERIMENT FOR BRASS SHIM STOCK MATERIAL (GALCIT TESTS)

$C_1 = 0.10$ $C_2 = 2.50$

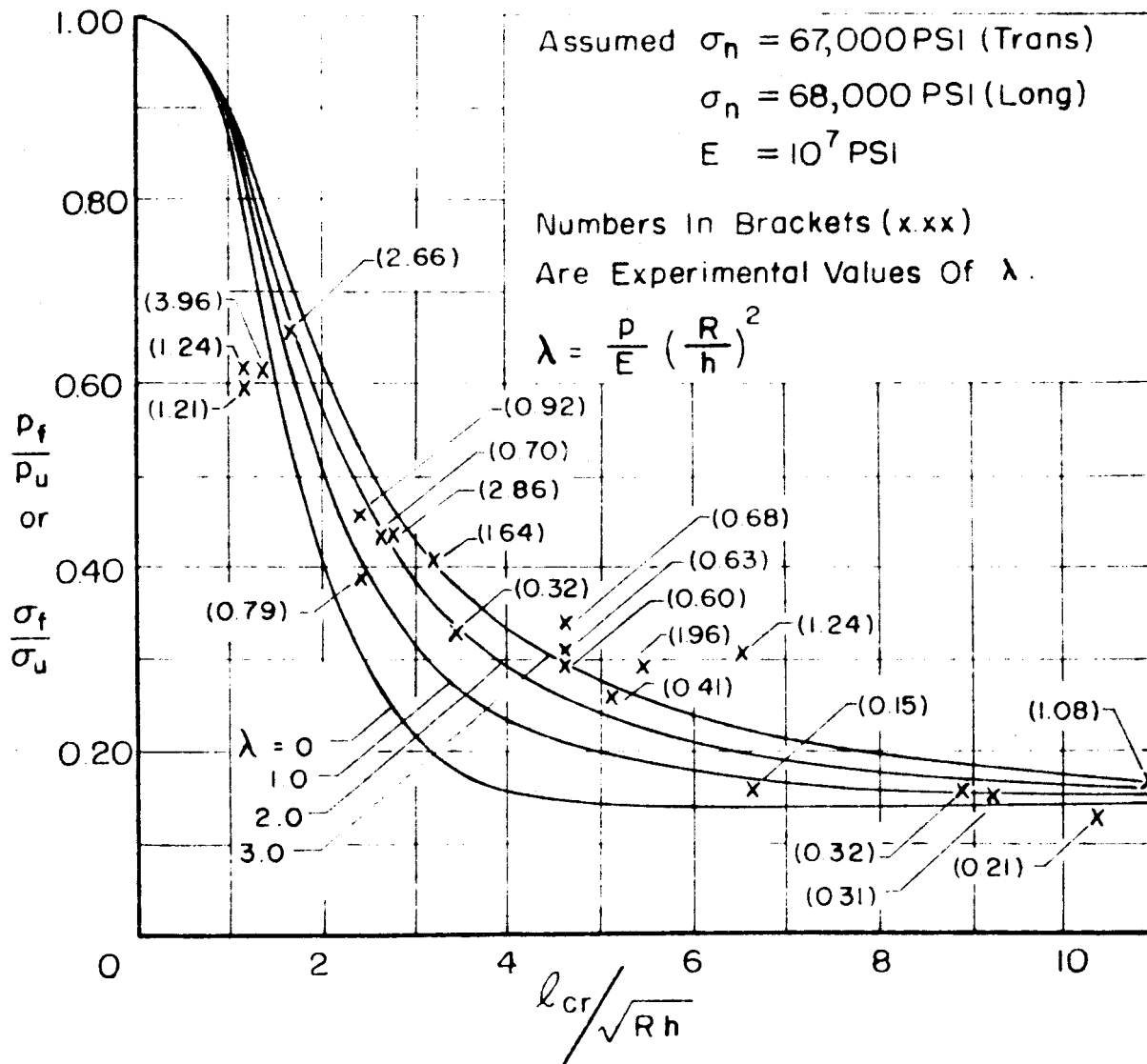


FIG. 5 - COMPARISON OF THEORY AND EXPERIMENT FOR
 2024-T3 MATERIAL (NACA TECH. NOTE 3993)
 $C_1 = 0.10$ $C_2 = 1.0$

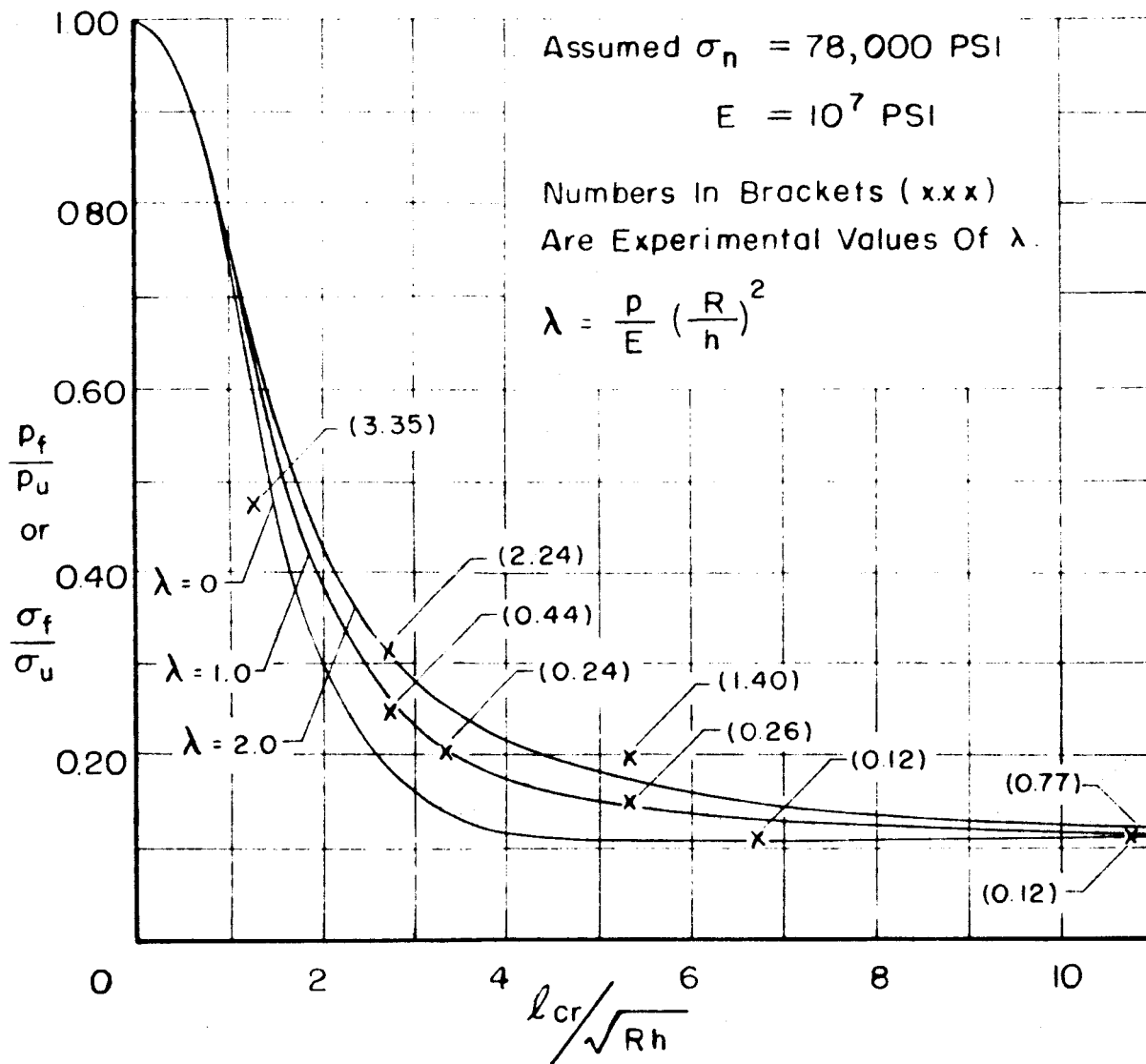


FIG. 6 - COMPARISON OF THEORY AND EXPERIMENT FOR
 7075-T6 MATERIAL (NACA TECH NOTE 3993)
 $C_1 = 0.10$ $C_2 = 0.75$

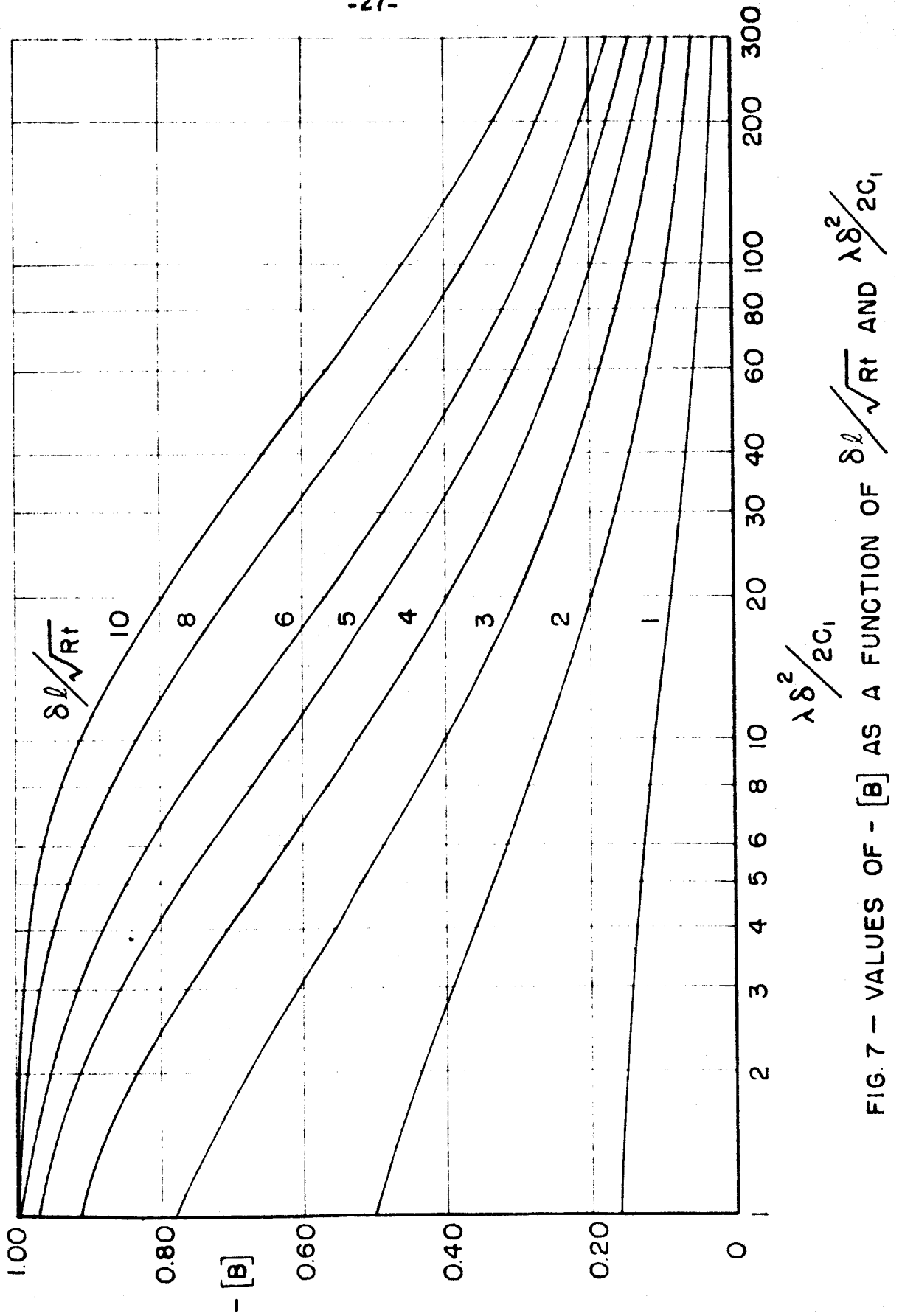


FIG. 7 - VALUES OF $-[B]$ AS A FUNCTION OF $\delta l/\sqrt{Rt}$ AND $\lambda\delta^2/2C_1$

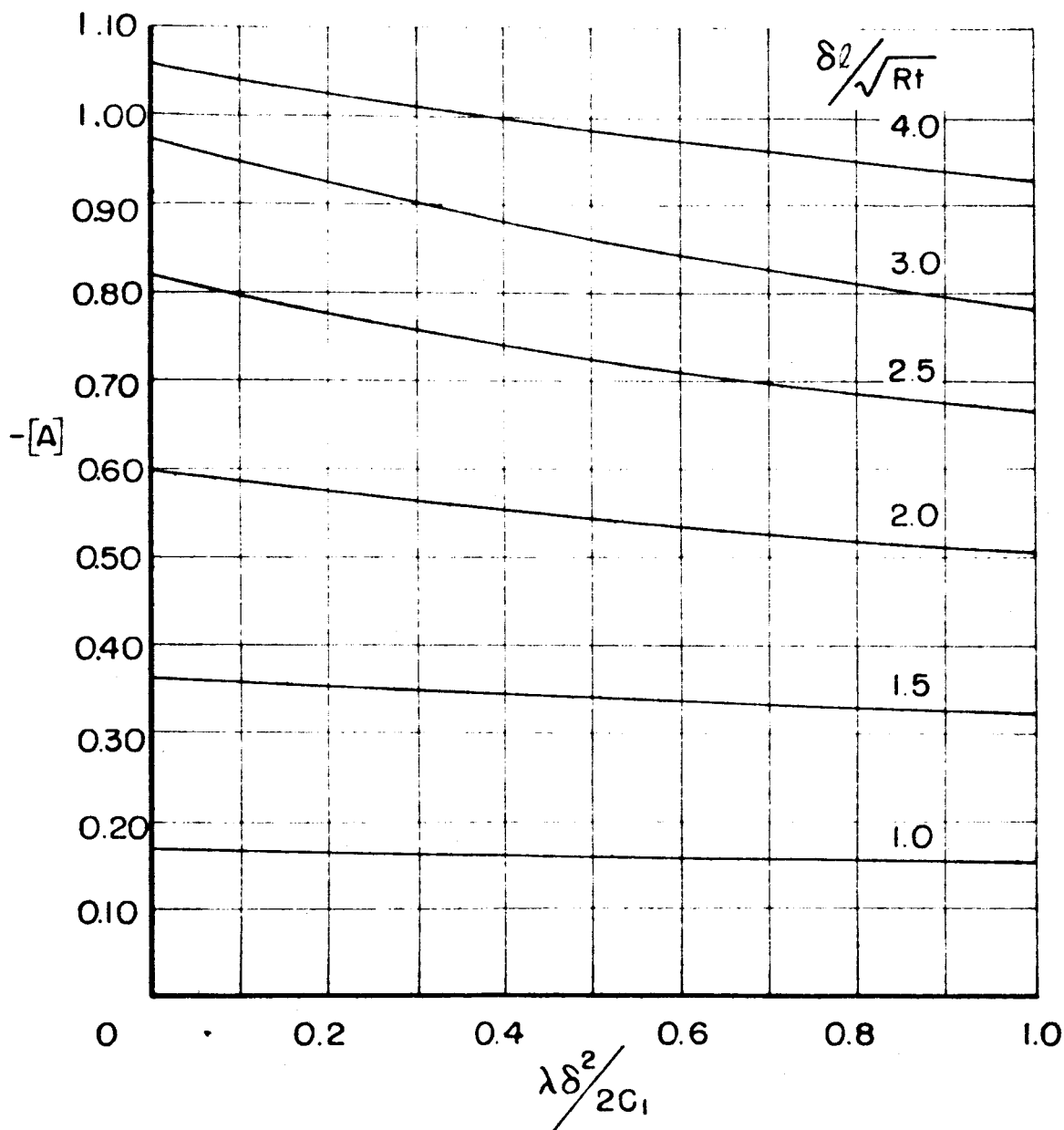


FIG. 8 - VALUES OF $[A]$ AS A FUNCTION OF $\frac{\delta l}{\sqrt{Rt}}$
 AND $\frac{\lambda \delta^2}{2C_1}$ FOR ALL VALUES OF $\frac{\delta l}{\sqrt{Rt}} > 4.0$,
 $[A] = -1.00$ WITHIN $\pm 4\%$, THE ERROR DECREASING
 FOR LARGER VALUES OF $\frac{\delta l}{\sqrt{Rt}}$

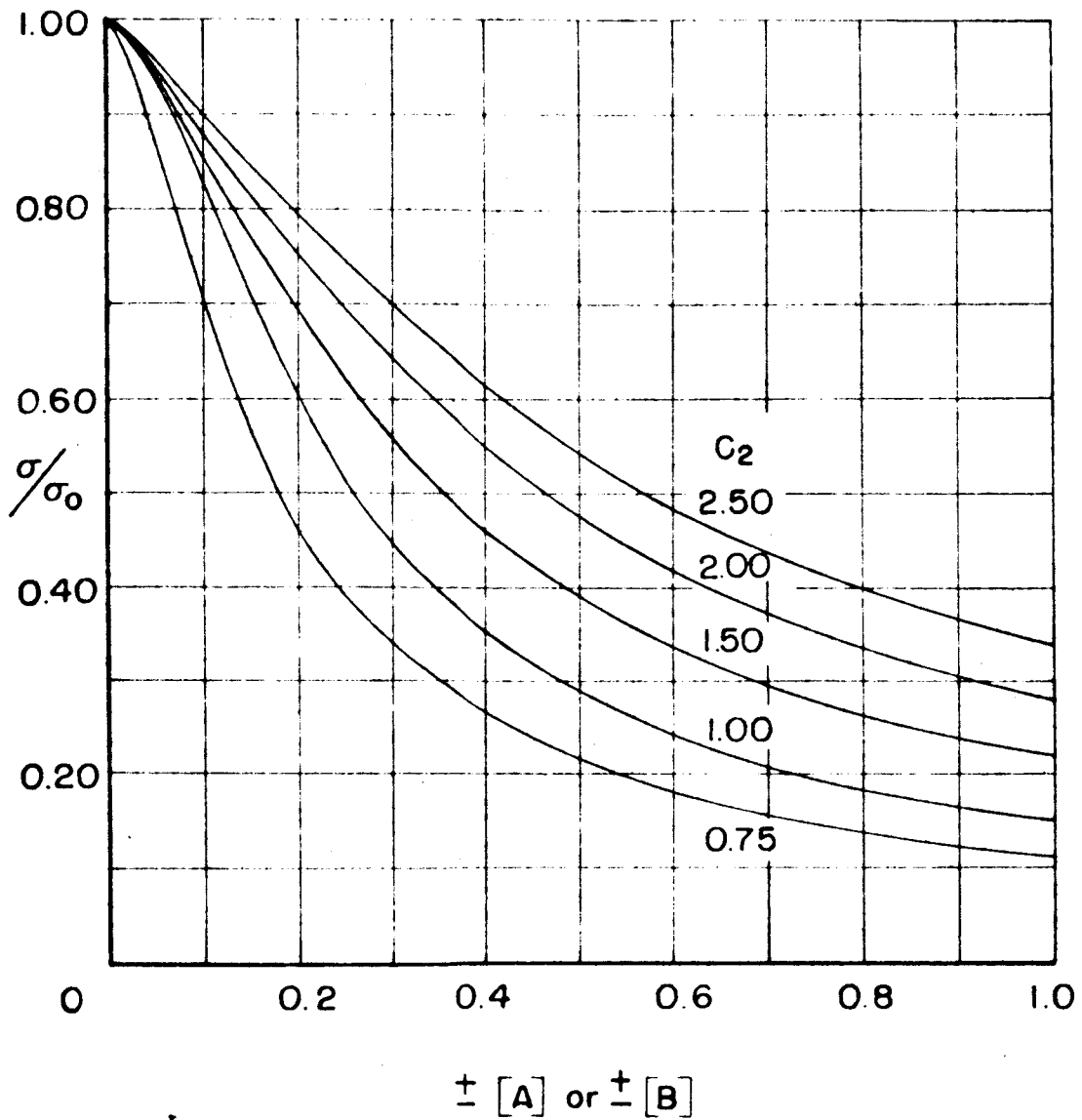


FIG. 9 - VALUES OF σ/σ_0 AS A FUNCTION OF $[]$
(EQUATION 45) AND C_2 FOR $C_1 = 0.10$